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First-order Modal and Temporal Logics: State of the art and perspectives

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Extended abstracts

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Introduction

The present booklet contains extended abstract of talks presented at the workshop *First-Order Modal and Temporal Logics: State of the Art and Perspectives* held on 7–11 Auguest 2023 as part of 34th European Summer School in Logic, Language and Information at the University of Ljubjana.

The workshop consisted of 5 invited and 7 contributed talks. Extended abstracts of all the contributed talks have been peer-reviewed by the members of the Programme Committee, listed on page 1 of this booklet.

Invited Talks

PROOFS WITH MODALITY: CHALLENGES AND PERSPECTIVES

BAHAREH AFSHARI

This talk is an invitation to revisit the proof theory of first-order modal and temporal logics in the light of advances in sequent calculi for modal and first order logic. Cyclic and ill-founded proofs provide a versatile grounding for the study of logics of recursive and/or co-recursive concepts. We survey some of the advantages offered by this notion of proof in the form of streamlined soundness and completeness arguments for modal and temporal logics and their applications to interpolation and proof search.

WHY PREDICATE ABSTRACTS (AND HOW)

MELVIN FITTING

Bertrand Russell (eventually) realized that definite descriptions need their scopes made explicit. In modal logics, which can have also have non-rigid constants, this is especially the case. Machinery for all this exists, predicate abstraction is one version, but it is not as well-known as it should be. I will present syntax and semantics suitable for this. Time won't permit discussion of corresponding tableau systems. But there should be enough material to get across the general need for predicate abstracts, and the general ideas of how they behave.

ADMISSIBLE SEMANTICS FOR QUANTIFIED MODAL AND TEMPORAL LOGICS

ROBERT GOLDBLATT

The term *admissible semantics* refers to the use of possible-worlds models having a restriction on which sets of worlds are admissible as propositions. Such models have proven effective in characterising propositional modal logics that are incomplete for their Kripke frame semantics.

There are axiomatically defined systems of quantified modal logic that cannot be characterised by the kind of possible-worlds models introduced by Kripke, even though the propositional fragments of those logics *are* characterised by their Kripke frames. We will describe how that this failure of completeness under Kripke semantics to lift from the propositional to the quantificational level can be overcome by developing a suitable notion of admissible model for quantified modal logics, leading to semantic characterisations of such logics in general. This requires a new interpretation of universal and existential quantifiers that takes into account the admissibility of propositions. The talk will explain the motivation for this interpretation.

It will also discuss an application to temporal logic. It has been known since the 1960's that temporal predicate logic over the real time flow is not recursively axiomatisable. What then of the axiom system that combines the standard deductive machinery of first-order logic with that of the temporal *propositional* logic of real time? It transpires that this system is strongly complete for validity in all *admissible* models over real time.

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DEFINITIONS AND (UNIFORM) INTERPOLANTS IN FIRST-ORDER MODAL LOGIC

AGI KURUCZ , FRANK WOLTER, AND MICHAEL ZAKHARYASCHEV

We consider some decidable fragments of first-order modal logics that do not enjoy the Craig interpolation or projective Beth definability properties, and so the existence of interpolants and explicit definitions of predicates do not directly reduce to the validity problem. Our concern is the computational complexity of deciding whether (uniform) interpolants and definitions exist for given input formulas.

COMPLETENESS IN FIRST-ORDER MODAL LOGIC: THE PRESENT STAGE

VALENTIN SHEHTMAN

The talk will give an overview of completeness and incompleteness results in first-order modal logic obtained in the 21st century. The situation can be briefly characterized as follows: completeness is unpredictable for Kripke and Kripke sheaf semantics, often expectable for simplicial semantics and usually unknown for intermediate semantics (Kripke bundle, functorial, Kripke metaframe). We will also discuss operations on logics and frames preserving completeness and the construction of Kripke completion.

Contributed Talks

EXPRESSING GLOBAL SUPERVENIENCE IN INQUISITIVE MODAL LOGIC

IVANO CIARDELLI

Introduction. Inquisitive logic [4] is an approach to logic which allows us to handle in a uniform way not only formulas regimenting statements, but also formulas regimenting various kinds of questions. For example, for every formula α of propositional or predicate logic, we will also have a corresponding formula $?\alpha$ representing the yes/no question *whether* α. Adding modalities to inquisitive logics we obtain *inquisitive modal logics*, conservative extensions of standard modal logics in which modalities may be applied to questions. E.g., in the context of a standard Kripke model, we can interpret not only a modal statement of the form $\Box p$ (where p is atomic), which has the usual interpretation, but also a modal statement of the form $\Box ?p$, which expresses the fact that all successors agree on the truth value of p. This extension of $\Box$ to questions has been studied so far in the setting of propositional modal logic [7, 2]; in this context, the possibility of applying $\Box$ to questions does not add to the expressive power of standard modal logic: for instance, $\Box ?p$ is equivalent to $\Box p \vee \Box \neg p$; more generally, every modal formula of the form $\Box \mu$ where μ is a question can be turned into an equivalent formula of standard modal logic.

In this talk, we will see that the situation changes when we turn to the setting of modal *predicate* logic. In this context, there are formulas of the form $\Box \mu$, where μ is an inquisitive formula, that are not equivalent to any formula of standard modal predicate logic. Moreover, some such formulas express very interesting modal facts. In particular, we will see that by adding $\Box$ to inquisitive first-order logic, we are able to express the *global supervience* of certain properties on others, i.e., the fact that the extension of the former is functionally determined, within the given modal range, by the extension of the latter. As an example, if P and Q are unary predicates, we may say that Q globally supervenes on P at world w in case:

$$\forall v, v' \in R[w] : P_v = P_{v'} \text{ implies } Q_v = Q_{v'}$$

where $R[w] = \{v \mid wRv\}$ and P_v is the extension of P at world v (similarly for $P_{v'}, Q_v, Q_{v'}$). We will show that this property is not expressible in standard modal predicate logic, but it is expressible by a simple modal formula in inquisitive modal predicate logic. This illustrates how, in the predicate logic setting, allowing $\Box$ to apply to questions increases the expressive power of modal predicate logic in an interesting way. We then turn to the properties of the resulting modal logic. We will then show how a broad fragment of our inquisitive modal logic allows for a kind of standard translation to classical first-order logic; as a consequence, the set of validities in this fragment is recursively enumerable, and the entailment relation is compact. (This not obvious, since it is an open problem whether these properties hold for the full language of inquisitive first-order logic, even without modalities.) Interestingly, this fragment includes all modal formulas expressing global supervenience claims.

Global supervenience. Supervenience claims are at the heart of many key discussions in analytic philosophy (see [10] for an overview). The general idea behind the notion of supervenience is as follows: given two classes of properties A and B, B supervenes on A if there cannot be a difference with respect to B-properties without a corresponding difference in A-properties. This informal idea can be made precise in different ways. One understanding focuses on individuals: two individuals cannot differ in their B-properties without also differing in their A-properties; this leads to various notions of *individual* supervenience (see [8]). Another understanding focuses on worlds as a whole: two worlds cannot differ in the extension of the B-properties without also differing in the extension of the A-properties. This leads to a notion of global supervenience, which in the context of a constant-domain Kripke model can be characterized as follows:[1]

$$\begin{array}{lll} P_1, \dots, P_n \rightsquigarrow_w Q_1, \dots, Q_m & \iff & \forall v, v' \in R[w] : \\ & & \text{if } (P_1)_v = (P_1)_{v'} \text{ and } \dots \text{ and } (P_n)_v = (P_n)_{v'} \\ & & \text{then } (Q_1)_v = (Q_1)_{v'} \text{ and } \dots \text{ and } (Q_n)_v = (Q_n)_{v'} \end{array}$$

[1] It should be noted that there are other notions of global supervenience, which are designed to apply in a setting where domains are not constant across worlds (see among others [12, 1, 9]). While most of the literature on global supervenience has focused on these other notions, the version that we identify seems to be an eminently natural way to cash out the idea of global supervenience in a constant-domain setting, and it is indeed the original characterization of global supervenience to be found in Kim's [8].

If the above relation holds, we say that $Q_1, \dots, Q_m$ globally supervene on $P_1, \dots, P_n$ in world w. We refer to $Q_1, \dots, Q_m$ as the supervenient properties and to $P_1, \dots, P_n$ as the subvenient properties. For simplicity, we will focus on the case of a single supervenient property and a single subvenient property, in which case the relation amounts to:

$$P \rightsquigarrow_w Q \quad \iff \quad \forall v, v' \in R[w] : P_v = P_{v'} \text{ implies } Q_v = Q_{v'}$$

However, our discussion extends straightforwardly to the general case.

Global supervenience is not definable in modal predicate logic. Consider standard modal predicate logic, QML, interpreted over first-order Kripke models with constant domains. We claim that there is no formula α of QML such that $M, w \models \alpha \iff P \rightsquigarrow_w Q$.

To prove this, we give a model that contains a pair of worlds w_0, w_1 which agree on all formulas of QML, and yet global supervenience holds in w_1 but not in w_0. Our model has the set $\mathbb{N}$ of natural numbers as its domain. Let E be the sets of even numbers and

$$\mathcal{X} = \{X \subseteq \mathbb{N} \mid X \text{ contains finitely many even numbers and all but finitely many odd numbers}\}$$

The universe of possible worlds includes, in addition to w_0 and w_1, worlds of the form v_{Xi} where $X \in \mathcal{X}$ and $i \in \{0, 1\}$: at world v_{Xi}, the extension of P is X, while the extension of Q is either $\emptyset$ of $\mathbb{N}$ depending on the Boolean value i:

$$P_{v_{Xi}} = X \qquad Q_{v_{Xi}} = \begin{cases} \mathbb{N} & \text{if } i = 1 \\ \emptyset & \text{if } i = 0 \end{cases}$$

At worlds w_0 and w_1, both extensions are empty. Next, we define a function $n : \mathcal{X} \to \{0, 1\}$ as follows: $n(X) = 0$ if $\#(X \cap E)$ is even, and $n(X) = 1$ if $\#(X \cap E)$ is odd (note that $X \cap E$ is finite by definition of $\mathcal{X}$). The accessibility relation is then defined as follows:

- $R[w_0] = \{v_{Xi} \mid X \in \mathcal{X} \text{ and } i \in \{0, 1\}\}$;
- $R[w_1] = \{v_{Xi} \mid X \in \mathcal{X} \text{ and } i = n(X)\}$;
- $R[v] = \emptyset$ for any world v distinct from w_0, w_1.

We have $P \rightsquigarrow_{w_1} Q$ but not $P \rightsquigarrow_{w_0} Q$. To see that the supervenience holds in w_1, suppose v_{Xi} and v_{Yj} are two successors of w_1 that assign the same extension to P; then $X = Y$, and so by the definition of $R[w_1]$ we have $i = n(X) = n(Y) = j$, which implies that the extension of Q is the same in v_{Xi} as in v_{Yj}. However, we do not have $P \rightsquigarrow_{w_0} Q$: indeed, for any $X \in \mathcal{X}$, the worlds v_{X0} and v_{X1} are both successors of w_0, and they assign the same extension X to P, but they disagree on the extension of Q.

At the same time, w_0 and w_1 are indistinguishable by formulas of QML, since they are bisimilar (for the notion of bisimulation in the setting of QML, see [13]). To argue for this, think in terms of a bisimulation game between two players, Spoiler and Duplicator. We sketch a winning strategy for Duplicator in the game starting from w_0 and w_1. In the first part of the game, until Spoiler picks an element from the domain $\mathbb{N}$, Duplicator responds with the same element. Now suppose at some point Spoiler decides to pick a successor of w_0 or w_1. The only interesting case to consider is the one where Spoiler picks a world $v_{Xi} \in R[w_0]$ such that $i \neq n(X)$ (in any other case, Duplicator can respond simply by picking the same successor). In that case, Duplicator picks v_{Yi} where $Y = X \cup \{h\}$ for some even number h which is different from any number in X and from any number picked so far in the game (this is possible since $X \cap E$ is finite). Note that $\#(Y \cap E) = \#(X \cap E) + 1$ and therefore, since $i \neq n(X)$, we have $i = n(Y)$, which implies that indeed $v_{Yi} \in R[w_1]$, as required. At this point, since the worlds v_{Xi} and v_{Yi} that have been reached do not have any successors, the only thing Spoiler can do is to pick an object on either side. Suppose $(a_1, \dots, a_m)$ and $(b_1, \dots, b_m)$ are the tuples picked so far. If Spoiler picks a_{m+1}, Duplicator may always pick b_{m+1} in such a way that (i) $a_{m+1} \in X \iff b_{m+1} \in Y$; (ii) if $a_{m+1} = a_j$ for some $j \leq m$ then $b_{m+1} = b_j$, while if a_{m+1} is distinct from all the previous a_j, then b_{m+1} is distinct from all the previous b_j. It is possible to achieve both (i) and (ii), since $Y \in \mathcal{X}$ guarantees that both Y and $\mathbb{N} - Y$ are infinite, and so we may always pick a fresh element in either of them. The argument is analogous if Spoiler picks b_{m+1}. It is easy to check that this is indeed a winning strategy for Duplicator.[2]

Global supervenience in inquisitive modal logic. We consider a system $\mathsf{InqQML}_\Box$ of inquisitive modal logic obtained by adding a modality $\Box$ to inquisitive first-order logic [4]. The language is given by the following definition:

$$\varphi \ := \ Rx_1 \dots x_n \mid x_1 = x_2 \mid \bot \mid \varphi \wedge \varphi \mid \varphi \to \varphi \mid \forall x \varphi \mid \Box \varphi \mid \varphi \vee\!\!\!\vee \varphi \mid \exists\!\!\!\exists x \varphi$$

As customary in inquisitive logic, we define $\neg\varphi := (\varphi \to \bot)$, $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$, $\exists x \varphi := \neg \forall x \neg \varphi$ and $?\varphi := \varphi \vee\!\!\!\vee \neg\varphi$. The operators $\vee\!\!\!\vee$ and $\exists\!\!\!\exists$ are called *inquisitive disjunction* and *inquisitive existential*

[2]The idea for this proof was developed in collaboration with Gianluca Grilletti.

quantifier and regarded as question-forming operators. The fragment of the language without these operators can be identified with standard modal predicate logic QML, while the fragment including $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$ but not $\exists$ will be denoted $\mathsf{InqQML}_\Box^-$.

Models for $\mathsf{InqQML}_\Box$ are standard Kripke models with constant domains. However, following inquisitive semantics [6], the interpretation of $\mathsf{InqQML}_\Box$ takes the form of a recursive definition of a relation $s \models_g \varphi$ called *support*, that holds between a set of worlds s, called an *information state*, and a formula φ (relative to an assignment g). A relation of truth at a world is retrieved by defining $w \models_g \varphi$ as a shorthand for $\{w\} \models_g \varphi$. A formula φ is said to be *truth-conditional* if support for φ boils down to truth at each world, in the sense that for every model M, state s, and assignment g:

$$s \models_g \varphi \iff \forall w \in s : w \models_g \varphi$$

Thus, if a formula is truth conditional then its semantics is completely determined by its truth conditions. The semantic clauses for atoms, connectives, and quantifiers are the standard ones from inquisitive logic (see [4]). As for formulas of the form $\Box\varphi$, they are stipulated to be truth-conditional with the following truth conditions:

$$w \models_g \Box\varphi \iff R[w] \models_g \varphi$$

It is easy to check that every formula α of standard modal logic (i.e., without $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$ or $\exists$) is truth-conditional, and its truth conditions coincide with the ones given by standard Kripke semantics. On the other hand, formulas involving inquisitive connectives are not in general truth-conditional. In particular, the formula $\forall x?Px$ is supported in s just in case the extension of P is the same in all worlds in s:

$$s \models \forall x?Px \iff \forall v, v' \in s : P_v = P_{v'}$$

Using this fact and the semantic clauses, it is easy to check that the modal formula $\Box(\forall x?Px \to \forall x?Qx)$ is true at a world just in case Q globally supervenes on P:

$$w \models \Box(\forall x?Px \to \forall x?Qx) \iff P \rightsquigarrow_w Q$$

Thus, in $\mathsf{InqQML}_\Box$ we have a (truth-conditional) formula that expresses the global supervenience of Q on P. Given the results in the previous section, this implies that the formula $\Box(\forall x?Px \to \forall x?Qx)$ is not equivalent to any formula of QML. Thus, in contrast to the propositional case, in the predicate logic setting allowing $\Box$ to apply to questions allows us to express (interesting) modal properties that standard modal logic cannot express.

More generally, the claim that $Q_1, \dots, Q_m$ supervene on $P_1, \dots, P_n$ is expressed by:

$$\Box(\forall x?P_1x \land \dots \land \forall x?P_nx \ \to \ \forall x?Q_1x \land \dots \land \forall x?Q_mx)$$

Two remarks: first, formulas expressing global supervenience relations do not contain $\exists$, and so they are in the fragment $\mathsf{InqQML}_\Box^-$. Second, such formulas take the form of strict conditionals $\Box(\varphi \to \psi)$ whose antecedent and consequent are questions; more specifically, the antecedent is the question asking for the extension of the subvenient properties, while the consequent is the question asking for the extension of the supervenient properties.[3]

Meta-theoretic properties of $\mathsf{InqQML}_\Box^-$. We will show that $\mathsf{InqQML}_\Box^-$, the $\exists$-free fragment of our logic, retains the key meta-theoretic properties of first-order logic. To show this, we build on ideas developed in [11] and [5]. First, we show that we can inductively define for each formula φ of $\mathsf{InqQML}_\Box^-$ a number n_φ such that φ is n_φ-*coherent* in the sense that the following holds for any M, s, g:

$$M, s \models_g \varphi \iff \forall t \subseteq s \text{ with } \#t \leq n_\varphi : M, t \models_g \varphi$$

Second, we define a family of translations from $\mathsf{InqQML}_\Box^-$ to a suitable two-sorted first-order language equipped with world variables. In particular, for any finite set $\mathsf{s} = \{\mathsf{w}_1, \dots, \mathsf{w}_n\}$ of world variables, we define a corresponding translation $\mathrm{tr}_\mathsf{s}(\varphi)$ which behaves semantically like φ on information states of size up to the number n of world variables in s. We then show that for any set $\Phi \cup \{\psi\}$ of formulas from $\mathsf{InqQML}_\Box^-$, if s is a set consisting of n_ψ-many world variables, we have:

$$\Phi \models_{\mathsf{InqQML}_\Box^-} \psi \iff \mathrm{tr}_\mathsf{s}(\Phi) \models_{\mathsf{FOL}} \mathrm{tr}_\mathsf{s}(\psi)$$

where on the left we have entailment in $\mathsf{InqQML}_\Box^-$ and on the right entailment in classical (two-sorted) first-order logic. Using this connection, it is then easy to show that $\mathsf{InqQML}_\Box^-$ is *entailment-compact* (i.e., $\Phi \models \psi$ implies $\Phi_0 \models \psi$ for some finite $\Phi_0 \subseteq \Phi$) and the set of theorems of $\mathsf{InqQML}_\Box^-$ is recursively enumerable. In sum, we will argue that $\mathsf{InqQML}_\Box^-$ provides a natural extension of standard modal

[3]In the setting of propositional modal logic, inquisitive strict conditionals have been studied in [3].

predicate logic that, among other things, allows us to regiment reasoning about global supervenience claims.

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AN ANDERSONIAN-KANGERIAN REDUCTION OF TERM-MODAL S5

STEF FRIJTERS

Term-modal logics (**TMLs**) are highly expressive first-order modal formalisms. They combine a full first-order language with modal operators indexed with terms (i.e. variables or constants) of the language. We denote these term-modal operators as $\Box_\theta$. The addition of such operators allows one to express complex sentences such as 'everyone believes that they are the hero of their own story' ($\forall x(\Box_x Hx)$). It was recently proven that many term-modal logics that do not validate the T-axiom are in fact fragments of standard (that is, not term-modal) first-order modal logics. It remained an open question whether the same could be proven for term-modal logics validating the T-axiom. In this talk we will partially answer this question by proposing a non term-modal logic **AKH** (based on some ideas taken from hybrid logic) of which the term-modal version of S5, **TMS5**, is a fragment.

Term-modal logics were first introduced by Fitting et al. [2] (for an overview, see [6]). We will present a simplified version of their semantics for **TMS5**, using constant instead of increasing domains. A **TMS5**-model is a tuple $M = \langle W, \mathcal{A}, R, I\rangle$, where W is a non-empty set of worlds and $\mathcal{A}$ is a non-empty set of agents. $R \subseteq W \times \mathcal{A} \times W$ is a ternary accessibility relation such that for every $w, w', w'' \in W$ and $p \in \mathcal{A}$: (1) $\langle w, p, w\rangle \in R$, (2) if $\langle w, p, w'\rangle \in R$, then $\langle w', p, w\rangle \in R$, and (3) if $\langle w, p, w'\rangle, \langle w', p, w''\rangle \in R$, then $\langle w, p, w''\rangle \in R$. Finally, I is an interpretation function assigning an element of $\mathcal{A}$ to every term and an element of $\wp(\mathcal{A}^n)$ to every pair consisting of an n-ary predicate and world $w \in W$. The semantic clauses are as usual, except that where θ is a term of the language, $M, w \models \Box_\theta\varphi$ iff $M, w' \models \varphi$ for all w' such that $\langle w, I(\theta), w'\rangle \in R$. See [3, 4] or below for more details.

Fitting et al. originally had an epistemic or doxastic reading of the term-modal operators in mind. Thus, $\Box_\theta\varphi$ is to be read as 'θ knows that φ' or 'θ believes that φ'. However, given the appropriate conditions on the accessibility relation R, there is nothing stopping us from giving the term-modal operator other readings. For example, in term-modal deontic logic (**TMDL**), $\Box_\theta\varphi$ is read as the personal obligation 'φ is obligatory for θ' [3, 4]. It is in this deontic context that the question arose whether **TMLs** can be reduced to standard (not term-modal) first-order modal logics.

It is a well-known result in deontic logic that many propositional deontic logics are reducible to, i.e. are fragments of, alethic modal logics. This is known as the Andersonian-Kangerian reduction of deontic logic. Anderson and Kanger proposed systems of alethic modal logic with a normative constant G, which can be read as 'what morality prescribes', 'a sanction is not applicable', or 'this is not a bad state of affairs'. They then defined $\mathsf{O}\varphi$, 'it is obligatory that φ', as $\Box(G \to \varphi)$: 'it is necessary for what morality prescribes that φ'. It has been proven that, for example, standard deontic logic (**SDL**) is a fragment of the Andersonian-Kangerian logic **K** extended with the axiom $\Diamond G$. In other words, one can define a translation from formulas of **SDL** to formulas of the Andersonian-Kangerian logic such that for every formula φ of **SDL**, φ is **SDL**-valid iff the translation of φ is valid in the Andersonian-Kangerian logic [3].

In [3] and forthcoming work, similar reduction results have been proven for a number of term-modal (deontic) logics that do not validate the T-scheme. The logics of which the term-modal logics are a fragment have some noteworthy properties. First, they do not contain a term-modal operator, but a standard modal operator. Secondly, these logics do not have propositional, but instead predicative constants in their language. For example, where $\mathcal{Q}$ is such a constant, $\mathcal{Q}\theta$ can be read as 'θ is a good person'. The formula $\Box_\theta\varphi$, 'it is obligatory for θ that φ', is then defined as $\Box(\mathcal{Q}\theta \to \varphi)$: 'it is necessary for θ being a good person that φ'. The proof for the reduction is significantly more complex than in the propositional case, but still follows the same basic outline.

Unfortunately, this approach does not seem generalizable to term-modal logics that validate the T-axiom, of which **TMS5** is an example. In this talk we propose a new logic, **AKH**, to solve this open problem. **AKH** combines the ideas of Andersonian-Kangerian logics with some ideas from hybrid logic.

The language of **AKH** extends that of first-order logic with a universal modal operator $[\mathsf{U}]$, a set of set variables $SV = \{X, Y, \ldots\}$ and a $\downarrow$ binder.[1] More precisely, the language is defined as follows.

[1] Note that the set variables $X, Y, \ldots$ are not predicate variables. As the semantics will show, their interpretation is more like that of the state variables in hybrid logic.

Let $C = \{a, b, \ldots\}$ be the set of constants and $V = \{x, y, \ldots\}$ be the set of variables. We let ν range over V. Let $T = C \cup V$ be the set of terms (always denoting persons) and $\theta, \theta_1, \ldots$ the metavariables ranging over it. For each natural number n we let $\mathcal{P}^n$ be a set of n-ary predicate symbols and we let $\mathcal{P}$ be the union of all $\mathcal{P}^n$. We let P range over $\mathcal{P}$. Let $SV = \{X, Y, \ldots\}$ be the set of set variables and let $\mathcal{X}$ be the meta-variable ranging over this set. Lastly, we let φ, ψ, χ be metavariables for formulas. Our language $\mathcal{L}$ is defined by the following Backus-Naur form:

$$\varphi ::= P\theta_1 \ldots \theta_n \mid \varphi \vee \varphi \mid \neg\varphi \mid [\mathsf{U}]\varphi \mid \forall\nu(\varphi) \mid \mathcal{X}^\theta \mid \downarrow\mathcal{X}^\theta(\varphi).$$

The semantics of **AKH** are given by the following definitions.

Definition 1 (Models)**.** *An* **AKH***-model is a tuple $M = \langle W, \mathcal{A}, f, I\rangle$ such that:*

1. *$W \neq \emptyset$ is the world-domain and $\mathcal{A} \neq \emptyset$ is the agent-domain*
2. *$f : \mathcal{A} \to \wp(\wp(W))$ is a function such that for every $p \in \mathcal{A}$, $f(p)$ is a partition of W, i.e. (1) for all distinct $\Gamma, \Delta \in f(p)$, $\Gamma \cap \Delta = \emptyset$, and (2) $\bigcup f(p) = W$*
3. *I is an* interpretation *function that assigns to every $\theta \in T$ a $p \in \mathcal{A}$ and to every pair $\langle P, w\rangle \in \mathcal{P}^n \times W$ an element of $\wp(\mathcal{A}^n)$ for every natural number $n \in \mathbb{N}$*

Definition 2 (Assignment function)**.** *An assignment function $g : SV \times \mathcal{A} \to \wp(W)$ on an* **AKH***-model $M = \langle W, \mathcal{A}, f, I\rangle$ is a function such that for every pair $\langle \mathcal{X}, p\rangle \in SV \times \mathcal{A}$, $g(\mathcal{X}, p) \in f(p)$.*

Definition 3 (ν-alternative)**.** *For any $\nu \in V$, $M' = \langle W, \mathcal{A}, f, I'\rangle$ is a ν-alternative to $M = \langle W, \mathcal{A}, f, I\rangle$ iff I' differs at most from I in the member of $\mathcal{A}$ that I' assigns to ν.*

Definition 4 ($\mathcal{X}, p$-alternative)**.** *$g_w^{\mathcal{X},p}$, the $\mathcal{X}, p$-alternative for g at w, is the function defined by letting $g_w^{\mathcal{X},p}(\mathcal{X}, p)$ be the unique $\Gamma \in f(p)$ such that $w \in \Gamma$ and letting $g_w^{\mathcal{X},p}(\mathcal{Y}, p') = g(\mathcal{Y}, p')$ for all $(\mathcal{Y}, p') \neq (\mathcal{X}, p)$.*

Definition 5 (Semantic clauses)**.** *Let $M = \langle W, \mathcal{A}, f, I\rangle$ be a model and let g be an assignment on M. Then we define:*

(SC1) $M, g, w \models P\theta_1 \ldots \theta_n$ *iff* $\langle I(\theta_1), \ldots, I(\theta_n)\rangle \in I(P, w)$
(SC2) $M, g, w \models \neg\varphi$ *iff* $M, g, w \not\models \varphi$
(SC3) $M, g, w \models \varphi \vee \psi$ *iff* $M, g, w \models \varphi$ *or* $M, g, w \models \psi$
(SC4) $M, g, w \models \theta = \kappa$ *iff* $I(\theta) = I(\kappa)$
(SC5) $M, g, w \models [\mathsf{U}]\varphi$ *iff* $M, g, w' \models \varphi$ *for all* $w' \in W$
(SC6) $M, g, w \models (\forall\nu)\varphi$ *iff for every ν-alternative M':* $M', g, w \models \varphi$
(SC7) $M, g, w \models \mathcal{X}^\theta$ *iff* $w \in g(\mathcal{X}, I(\theta))$
(SC8) $M, g, w \models \downarrow\mathcal{X}^\theta(\varphi)$ *iff* $M, g_w^{\mathcal{X},I(\theta)}, w \models \varphi$

The main idea behind the semantics is that for every $p \in \mathcal{A}$, the function f gives a partition of the world-domain, which corresponds with the partition induced by the accessibility relation in **TMS5**. Note that because of Definition 2, our set variables are different from the state variables usually employed in hybrid logic. In standard hybrid logic (see e.g. [1, p. 825]), every state variable is true at exactly one world, i.e. every state variable 'names' a world. In contrast, in our approach there is a partition of the set of worlds for every agent θ, and every X^θ 'names' a cell of this partition, i.e. X^θ is true in all and only the worlds in one cell of the partition. The intuition behind the $\downarrow$-operator is similar to that in hybrid logic. In hybrid logic the $\downarrow$-operator allows one to 'name' or reference the world (hence the original name *reference pointer*) [5, 1]. In **AKH**, the $\downarrow$-operator allows one to 'name' or reference the set of which the world is a part.

With this toolbox we can define the term-modal operator $\Box_\theta$ as:

$$\Box_\theta\varphi := \downarrow X^\theta([\mathsf{U}](X^\theta \to \varphi))$$

This definition is in the first place meant to be a technical definition to allow for the reduction. However, we can also give it a more intuitive reading. To do so, we stick to the epistemic reading of $\Box_\theta\varphi$, 'θ knows that φ'. Now several different readings of X^θ are possible. We can read X^θ as 'the total body of evidence of agent θ is called X'. Then 'θ knows that φ', $\Box_\theta\varphi$, is analyzed as 'if we call the total body of evidence that θ has (in this world) X, then every world where θ has exactly this body of evidence X makes φ true'. Shortened this becomes: 'θ's total body of evidence necessarily implies φ'. Alternatively, we could read X^θ as 'all that θ knows is X' or 'θ's knowledge base is called X'. There is a fruitful philosophical discussion to be had about the proper reading of X^θ.

The reduction proposed in this talk has other upshots as well. Firstly, **AKH** is more expressive than **TMS5**. For example, the **AKH**-formulas $\downarrow X^\theta([\mathsf{U}](X^\theta \leftrightarrow \varphi))$ and $\downarrow X^\theta([\mathsf{U}](\varphi \to X^\theta))$ do not have a counterpart in **TMS5**, but do formalise useful statements. Given the reading proposed above, the first formula formalises the statement 'θ's total body of evidence is (necessarily) equivalent to φ' and the second formalises 'φ necessarily implies θ's total body of evidence' (see also [7]). Secondly, the reduction of **TMDLs** showed that term-modal logics are fragments of first-order modal logic, and thus not as exotic as they might seem at first. The fact that this simple reduction does not seem to work for **TMS5** was surprising. Perhaps even more surprising is the fact that we seem to need a highly unorthodox logic like **AKH** to reduce **TMS5** to a non term-modal logic. This deserves further investigation. Other possible paths of further research are reductions for **TMLs** with variable domain semantics, or for **TMLs** that validate the T-axiom but are weaker than **TMS5**.

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THE PREDICATE MODAL LOGIC OF FORCING

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Abstract. We report on research in progress, supervised by Joel David Hamkins and Benedikt Löwe. Hamkins and Löwe determined the (propositional) modal logic of forcing to be S4.2; we aim to determine the predicate modal logic of forcing.

Forcing is a fundamental technique in set theory that was introduced in 1963 by Paul Cohen and was first used to prove the independence of the Continuum Hypothesis in [1]. It is a method for constructing new models of set theory by extending an already known model, the ground model, in a carefully chosen way as to allow for a considerable amount of control over the structure and truths of the extension model. The technique has revolutionized the field of set theory, leading to far-reaching applications and an abundance of new models of ZFC.

This relation between a ground model and its forcing extensions has led to the notion of the set-theoretic multiverse, a rich and complex hierarchy of set-theoretic universes. Its structure has been studied by means of a forcing interpretation of the modalities $\Box$ and $\Diamond$. For a model $\mathcal{M}$ of set theory we interpret $\mathcal{M} \models \Box\varphi$ as "in every forcing extension φ holds" and $\mathcal{M} \models \Diamond\varphi$ as "in some forcing extension φ holds". Further, we say that $\psi(p_0, ..., p_n)$ is a ZFC-provable propositional modal principle of forcing if it is a propositional modal sentence such that $\psi(\varphi_0, ..., \varphi_n)$ is provable for all set-theoretic sentences $\varphi_0, ..., \varphi_n$. The forcing interpretation of $\Box$ and $\Diamond$ was first introduced by Hamkins in [11], where the relative consistency of ZFC together with the maximality principle $\Diamond\Box p \to \Box p$ was shown. Subsequently, in [12], a new area of research, the modal logic of forcing, was introduced by Hamkins and Löwe, and the propositional modal principles of forcing that are provable from ZFC were determined to precisely match the modal logic S4.2. This was followed by several works by various authors which further established the modal account on forcing, among them [11, 18, 16, 8, 9, 19, 21, 7, 6, 13, 10, 14, 23, 20]. The techniques developed for the study of modal logics of multiverses have been fruitfully used in other structural areas of mathematics, cf., e.g., [2, 15, 22, 3].

In the presentation, I shall report on an ongoing project to extend the results by Hamkins & Löwe to determine the *predicate modal logic of forcing*. More specifically, let $\mathcal{L}^\Diamond$ be the first-order modal language containing symbols for infinitely many predicates P_i of each arity and infinitely many variables $x, y, z, ...$, and let formulas of $\mathcal{L}^\Diamond$ be closed under Boolean connectives, modal operators and quantifiers. Where $\mathcal{L}^\in$ is the language of set theory, we can now define what it means to be a forcing translation.

Definition. A forcing translation is a function σ, mapping formulas ψ of $\mathcal{L}^\Diamond$ to formulas ψ^σ of $\mathcal{L}^\in$, defined recursively as follows, where the φ_i are $\mathcal{L}^\in$ formulas with as many free variables as the arity of the respective predicates P_i.

$$\begin{aligned}
P_i(\bar{x})^\sigma &\equiv \varphi_i(\bar{x}) \\
(\psi_0(\bar{x}) \wedge \psi_1(\bar{y}))^\sigma &\equiv \psi_0(\bar{x})^\sigma \wedge \psi_1(\bar{y})^\sigma \\
(\neg\psi(\bar{x}))^\sigma &\equiv \neg\psi(\bar{x})^\sigma \\
(\forall x \psi(x, \bar{y}))^\sigma &\equiv \forall x\ \psi(x, \bar{y})^\sigma \\
(\Box\psi(\bar{x}))^\sigma &\equiv \text{ in every forcing extension } \psi(\bar{x})^\sigma
\end{aligned}$$

In other words, σ is a forcing translation if it maps ψ to a substitution instance of ψ where predicates P_i are replaced by formulas φ_i having the same number of arguments such that each instance of φ_i takes the free variables that P_i would have taken in the same instance in the original formula.

Definition. A predicate modal assertion ψ is a ZFC-*provable principle of forcing* if for all forcing translations σ, $\mathsf{ZFC} \vdash \psi^\sigma$.

The goal of our project is to determine the ZFC-provable predicate principles of forcing. An example of such a principle is the *converse Barcan formula*

$$\Box\forall x P(x) \to \forall x \Box P(x),$$

which is always valid in a Kripke model with inflationary domains. Indeed, if $\forall x\varphi(x)$ is true in every forcing extension, then in particular, $\varphi(x)$ will be true in every forcing extension for every set x in the ground model, precisely because x continues to exist in the extension. In fact, this formula is even provable from the axioms and rules of first-order logic together with those of the smallest normal modal logic K (hence is included in $\mathsf{QS4.2}$ below).

It turns out that the answer to our main question might differ considerably depending on whether or not our language contains equality: we conjecture the answer to be different for the cases *without* and *with* equality.

Definition. We let $\mathsf{QS4.2}$ be the smallest set of formulas containing

(1) axioms of first-order logic without equality and
(2) $\psi(\chi_0, ..., \chi_{n-1})$ whenever $\psi(p_0, ..., p_{n-1})$ is a formula of propositional $\mathsf{S4.2}$, where the χ_i are formulas of $\mathcal{L}^\Diamond$ without equality,

and closed under the rules Modus Ponens, Necessitation, Universal Instantiation and Universal Generalisation. Further, we let $\mathsf{QS4.2}^=$ be defined as above but include equality in both points (1) and (2).

Conjecture 1. The ZFC-provable principles of forcing *without equality* are exactly those sentences in $\mathsf{QS4.2}$.

Conjecture 2. The ZFC-provable principles of forcing *with equality* are exactly those sentences in the smallest set of formulas containing

(1) the formulas in $\mathsf{QS4.2}^=$,
(2) *Necessary Identity* (NI)
$$\forall x\forall y(x = y \iff \Box x = y),$$
(3) *Necessary Non-identity* (NNI)
$$\forall x\forall y(x \neq y \iff \Box x \neq y),$$
(4) and *Infinite Domains* (InfD), which is the set of sentences
$$\{\exists x_0 ... \exists x_n \bigwedge_{i\neq j} x_i \neq x_j \mid n \in \omega\},$$

and is closed under the rules Modus Ponens, Necessitation, Universal Instantiation and Universal Generalisation.

The approach we aim to follow in proving these conjectures is based on the method developed in [12] and further specified in [14]. As in the propositional case, the lower bounds (i.e., showing that every formula conjectured to be a provable principle is a provable principle) are easy to verify and we can readily do so. The upper bounds (i.e., showing that no other formulas are provable principles) are considerably harder. In the propositional case, this is done by so-called *control statements* that we can determine for modal logics that are characterised by a class of finite frames (cf. [14, § 4]). Unfortunately, neither of the conjectured predicate modal logics have the finite frame property, so we must adjust this idea.

In the talk, I shall give some details of the techniques that we plan to employ to solve this technical problem and approach the proof of the two conjectures. This talk reports on work supervised by Joel David Hamkins and Benedikt Löwe.

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SATISFIABILITY PROBLEM FOR THE BUNDLED FRAGMENTS OF FIRST ORDER MODAL LOGIC

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In first-order modal logic (FOML), it is well-known that finding decidable fragments of FOML is hard. There are a very few fragments like the *monodic fragments* ([WZ01]) that are decidable. [1] When we bundle quantifiers and modalities together (as in $\exists x\Box$, $\Diamond\forall x$, etc.), we get new logical operators whose combinations produce interesting fragments of FOML without any restriction on the arity of predicates, the number of variables, or the modal scope. It has been shown that when the existential quantifier and a box modality were always bundled together to appear as a single quantifier-modality pair ($\exists x\Box$), the resulting fragment of FOML is decidable ([Wan17]). This fragment is motivated by epistemic operators that go beyond the classical know-that, and captures the logic of many *knowing-wh* expressions such as *knowing what, knowing how, knowing why*, and so on, e.g., *knowing how to achieve* φ is rendered as *there exists* a method x such that the agent *knows that* x can guarantee φ ([Wan18].

The motivation for 'bundling' is to restrict the occurrences of quantifiers using modalities. For instance, allowing only formulas of the form $\forall x\Box\alpha$ is one such bundling. On the other hand, we could also have $\Diamond\exists y\alpha$. Thus, there are many ways to 'bundle' the quantifiers and modalities. We call these the 'bundled operators/modalities'. The following syntax defines all possible bundled operators of one quantifier and one modality. Note that we exclude equality, constants, and function symbols from the syntax.

Definition 1 (Bundled-FOML syntax)**.** *Given a countable set of predicates $\mathcal{P}$ and a countable set of variables Var, the bundled fragment of* FOML *is the set of all formulas constructed by the following syntax:*

$$\alpha ::= P(x_1,\dots,x_n) \mid \neg\alpha \mid \alpha\wedge\alpha \mid \Box\alpha \mid \forall x\Box\alpha \mid \exists x\Box\alpha \mid \Box\forall x\alpha \mid \Box\exists x\alpha$$

where $P \in \mathcal{P}$ has arity n and $x, x_1, \dots, x_n \in \mathsf{Var}$.

We denote AB (to mean forAll-Box) to be the language that allows only atomic formulas, negation, conjunction, $\Box\alpha$ and $\forall x\Box\alpha$ (dually $\exists x\Diamond\alpha$) formulas. Similarly, we have EB(Exists-Box), BA(Box-forAll) and BE(Box-Exists) to mean the fragments that allows formulas of the form $\exists x\Box\alpha$, $\Box\forall x\alpha$ and $\Box\exists x\alpha$ and their duals respectively.

Definition 2 (FOML structure)**.** *An increasing domain model for* FOML *is a tuple $\mathcal{M} = (\mathcal{W}, \mathcal{D}, \delta, \mathcal{R}, \rho)$ where $\mathcal{W}$ is a non-empty countable set called* worlds*; $\mathcal{D}$ is a non-empty countable set called* domain*; $\mathcal{R} \subseteq (\mathcal{W}\times\mathcal{W})$ is the* accessibility relation*. The map $\delta : \mathcal{W} \mapsto 2^{\mathcal{D}}$ assigns to each $w \in \mathcal{W}$ a non-empty local domain set such that whenever $(w, v) \in \mathcal{R}$ we have $\delta(w) \subseteq \delta(v)$ and $\rho : (\mathcal{W}\times\mathcal{P}) \mapsto \bigcup_n 2^{\mathcal{D}^n}$ is the* valuation function*, which specifies the interpretation of predicates at every world over the local domain with appropriate arity. The model $\mathcal{M}$ is said to be a* constant domain model *if for all $w \in \mathcal{W}$ we have $\delta(w) = \mathcal{D}$. When $\delta(w) = \delta(v)$ for all $w, v \in \mathcal{W}$, we call $\mathcal{M}$ is a constant domain model.*

Definition 3 (FOML semantics)**.** *Given an* FOML *model $\mathcal{M} = (\mathcal{W}, \mathcal{D}, \delta, \mathcal{R}, \rho)$ and $w \in \mathcal{W}$, and σ relevant at w, for all* FOML *formulas α define $\mathcal{M}, w, \sigma \models \alpha$ inductively as follows:*

$\mathcal{M}, w, \sigma \models P(x_1,\dots,x_n)$	$\Leftrightarrow$	$(\sigma(x_1),\dots,\sigma(x_n)) \in \rho(w, P)$
$\mathcal{M}, w, \sigma \models \neg\alpha$	$\Leftrightarrow$	$\mathcal{M}, w, \sigma \not\models \alpha$
$\mathcal{M}, w, \sigma \models \alpha\wedge\beta$	$\Leftrightarrow$	$\mathcal{M}, w, \sigma \models \alpha$ *and* $\mathcal{M}, w, \sigma \models \beta$
$\mathcal{M}, w, \sigma \models \exists x\alpha$	$\Leftrightarrow$	*there is some* $d \in \delta(w)$ *such that* $\mathcal{M}, w, \sigma_{[x\mapsto d]} \models \alpha$
$\mathcal{M}, w, \sigma \models \Box\alpha$	$\Leftrightarrow$	*for every* $u \in \mathcal{W}$ *if* $(w, u) \in \mathcal{R}$ *then* $\mathcal{M}, u, \sigma \models \alpha$

Note that in bundled fragments such as EB, a modality comes right after a quantifier as in $\exists x\Box\varphi$, thus $\exists x(\Box\varphi \wedge \Diamond\psi)$ is not in the fragment EB whatever φ and ψ are. We may weaken this condition to allow formulas of the form $\exists x\beta$ where β is a boolean combination of atomic formulas and modal formulas. Moreover, we can allow a quantifier alternation of the form $\exists x_1\cdots\exists x_n\ \forall y_1\cdots\forall y_m\ \beta$. As a result, we obtain *loosely bundled fragment* (LBF):

[1]Monodic fragment requires that there be at most one free variable in the scope of any modal subformula.

Domain	$\forall\Box$	$\exists\Box$	$\Box\forall$	$\Box\exists$	Upper/ Lower Bound
Constant	✓	$\star$	$\star$	$\star$	Undecidable
	$\star$	$\star$	✓	$\star$	
	✗	✓	✗	✗	PSPACE-complete
	✗	✗	✗	✓	No FMP
	✗	✓	✗	✓	
Increasing	✓	✗	✗	✗	PSPACE-complete
	✗	✓	✗	✗	
	✗	✗	✓	✗	
	✗	✗	✗	✓	EXPSPACE/ PSPACE
	✓	✓	✗	✗	EXPSPACE/NEXPTIME
	✗	✗	✓	✓	
	$\star$	✓	✓	$\star$	Undecidable
	✗	✓	✗	✓	No FMP
	✓	✓	✗	✓	Undecidable
	✓	✗	✓	✓	EXPSPACE/ NEXPTIME
	loosely bundled				

FIGURE 1. Satisfiability problem classification for combinations of bundled fragments. ($\star$ means that the result holds with or without the presence of the corresponding bundle.)

Definition 4 (LBF syntax)**.** *The loosely bundled fragment of* FOML *is the set of all formulas constructed by the following syntax of* α*:*

$$\psi ::= \quad P(z_1, \ldots z_n) \mid \neg P(z_1, \ldots z_n) \mid \psi \wedge \psi \mid \psi \vee \psi \mid \Box\alpha \mid \Diamond\alpha$$
$$\alpha ::= \quad \psi \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \exists x_1 \ldots \exists x_k \forall y_1 \ldots \forall y_l \; \psi$$

where $k, l, n \geq 0$*,* $P \in \mathcal{P}$ *has arity* n *and* $x_1, \ldots x_k$*,* $y_1, \ldots y_l$*,* $z_1, \ldots, z_n \in \mathsf{Var}$*.*

Besides EB, the bundled fragment ABEB is still decidable over increasing domain models, though it was later shown that there was a price to be paid in terms of complexity ([PRW18]). This opens up a range of questions: what about other bundles, such as BE or BA and combinations thereof? Which of these distinguishes constant domain and increasing domain models? What about further bundles such as $\forall x\exists y\Box$ etc.? Can we identify the borderline between decidability and undecidability in this terrain? In [LPRW22] we consider all the bundles and classify them as: decidable ones, undecidable ones, and for those without definite answers yet, we show they lack the finite model property. Moreover, the LBF generalizes the bundling idea to what we believe to be the strongest yet decidable bundled fragment. The results are concluded in the Figure 1. Noted that constant domain and increasing domain interpretations make a significant difference.

We provide an informal guide to our latter results according to the expressivity of the bundled fragments. If a fragment can express, modulo some modal padding in a restricted way, both $\forall x\exists y\alpha$ and $\forall x\forall y\forall z\alpha$ in some form (like EBBA and ABEBBE), we can then prove that such a fragment is undecidable. If a fragment can express the essence of $\forall x\exists y\alpha$ but not $\forall x\forall y\forall z\alpha$ (like EBBE and BE over constant domain models) then we will prove that such fragments do not have finite model property. Finally, if a fragment cannot express the essence of $\forall x\exists y\alpha$ (like ABEBBA and LBF) then we will prove that it satisfies finite model property and give a tableau procedure.

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MEZHIROV'S GAME FOR INTUITIONISTIC LOGIC AND ITS VARIATIONS

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Game semantics allows us to look at basic logical concepts from another side. This approach to logic has a long history, there are plenty of different types of games: provability games, semantic games, etc [10,11]. And there is an interesting type of provability games called Mezhirov's game proposed by Iliya Mezhirov for intuitionistic logic of propositions (IPC) and Grzegorczyk modal logic (Grz) [1,2]. This idea was developed in many different directions; for example, in 2008 in the joint paper with N. Vereschagin a game semantics was given for affine and linear logic [3]. Independently G. Japaridze worked on game semantics for linear logic [4]. Mezhirov's games for minimal propositional logic (MPC), logic of functional frames ($KD!$) and logic of serial frames (KD) were introduced in 2021 by A. Pavlova [5].

Mezhirov's game semantics for intuitionistic logic is interesting because of its simplicity and strong connection with Kripke semantics and Kripke models. The game between Opponent and Proponent starts with a formula φ. And Proponent has a winning strategy iff φ is an intuitionistic tautology. The connection between the game and Kripke models manifests itself in building strategy for Opponent from a Kripke model (Opponent "walks" from one world of a model to another) and in the reconstruction of a model from Opponent's winning strategy (in which there exists a world where φ is false). And these procedures are connected to each other.

In my study, I try to generalize Mezhirov's result in two directions: to generalize to intuitionistic logic of predicates (introduce a game between Opponent and Proponent with at least the same connection with Kripke models or with special classes of them) and to the case of a connection not only between the game and tautologies of logic ($\vDash \varphi$), but also between the game and entailment from infinite sets of formulas ($T \vDash \varphi$).

The purpose of building such game was to get a theorem of kind "Proponent has a winning strategy in a special starting position (easily defined using an arbitrary set of formulas $\mathcal{O}_0$ and formula φ) iff $\mathcal{O}_0 \vDash \varphi$", where $\vDash$ is the semantic consequence defined by some class of predicate Kripke frames (φ is a semantic consequence of T in some class C of predicate Kripke frames iff for each Kripke model, based on a frame from the class C, if all formulas from T is true everywhere in this model, then φ is true in each world of the model). I initially thought about just logic of all Kripke models, i.e. it would be a game for intuitionistic logic of predicates directly. But it turned out that in such case some fundamental problems arise and it is natural to expand the logic (to use a smaller class of Kripke frames). Moreover, description of such variations (not just logic of all Kripke models) could be useful, since, in general, Kripke semantics for superintuitionistic predicate logic is rather weak (e.g. [9]). And I managed to get a description (based on the game I built) for several variations. So let me describe the rules of the game.

Let Ω be the elementary intuitionistic language (without function symbols; language will contain $\perp$, and the set of logical connectives will be $\{\to, \wedge, \vee\}$, where $\neg A$ will be considered as $A \to \perp$), and we will use Kripke models for intuitionistic logic of predicates [6,7] (I will call sets of constants in each world "individual domains" (or "the set of objects") and use symbol Δ). For the set of formulas Γ and set of objects (constants) Δ let $\mathcal{F}(\Gamma, \Delta) = \{P[c_1, ..., c_n] | P[x_1, ..., x_n]$ is a subformula of some formula from Γ and free variables of it are only $x_1, ..., x_n; c_i \in \Delta\}$ (so $\mathcal{F}$ in some ways is a set of all "subformulas" of formulas from Γ). Players Opponent and Proponent will be associated with their sets $\mathcal{O}$ and $\mathcal{P}$. The position in the game is a triple $\mathcal{C} = (\mathcal{O}, \mathcal{P}, \Delta)$. In each position $\mathcal{C}$: $\mathcal{O}$ and $\mathcal{P}$ are subsets of $\mathcal{F}(\Gamma, \Delta)$, where Δ is taken from $\mathcal{C}$ (and can only expand in the game process) and Γ is fixed at the beginning of the game and does not change until the end and equals to $\mathcal{O}_0 \cup \{\varphi\}$ (where $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ is a starting position; Δ_0 is an exact set of all constants contained in formulas from Γ). Proponent moves by adding new formulas from $\mathcal{F}$ to $\mathcal{P}$, Opponent moves by expanding Δ (he can add nothing to Δ if he wants; and he can add to Δ not just constants from Ω) and than adding new formulas from $\mathcal{F}$ to $\mathcal{O}$.

The only thing left to define is who must move in a position $\mathcal{C}$. To do that, let us firstly define the notion of truth relation $\Vdash$ in $\mathcal{C}$ for formulas from $\mathcal{F}(\Gamma, \Delta)$:

$$\begin{array}{l} \mathcal{C} \nVdash \bot \\ \mathcal{C} \Vdash A[c_1, ..., c_n] \rightleftharpoons A[c_1, ..., c_n] \in \mathcal{O} \\ \mathcal{C} \Vdash \varphi \star \psi \rightleftharpoons \varphi \star \psi \in \mathcal{O} \cup \mathcal{P} \text{ and } (\mathcal{C} \Vdash \varphi) \star (\mathcal{C} \Vdash \psi),\ \star \in \{\rightarrow, \wedge, \vee\} \\ \mathcal{C} \Vdash qxP[x] \rightleftharpoons qxP[x] \in \mathcal{O} \cup \mathcal{P} \text{ and } q\alpha \in \Delta(\mathcal{C} \Vdash P[\alpha]),\ q \in \{\exists, \forall\} \end{array}$$

where A is a predicate symbol, $arity(A) = n$, $c_i \in \Delta$, P - formula with only one free variable. A star in the case of $(\mathcal{C} \Vdash \varphi) \star (\mathcal{C} \Vdash \psi)$ means logical meta connective and behaves like a classical connective (the same for q in $q\alpha \in \Delta$).

Let us call a formula from $\mathcal{P}$ Proponent's mistake if it is false in the current position (the same for $\mathcal{O}$ and Opponent). If Opponent has no mistakes but Proponent has, then Proponent moves. Otherwise, Opponent must move. And if after a turn of a fixed player he must move again, he loses. If the game goes on infinitely (each player manages to pass a turn to the other player each turn), Proponent wins; also let us call formulas from $\mathcal{O} \cup \mathcal{P}$ marked formulas.

Now let us consider several examples of the game. In the first game $\mathcal{C}_0 = (\varnothing, \{\varphi\}, \varnothing)$, where $\varphi = \forall y \exists x (P[x] \rightarrow P[y])$. Because Δ is empty, there are no formulas in $\mathcal{F}$ of the form $\exists x(P[x] \rightarrow P[c])$, so Proponent has no mistakes, it's Opponent's turn. It is enough for him to just expand Δ, and it will be Proponent's turn. Proponent takes all formulas of the kind $\exists x(P[x] \rightarrow P[c])$ and $P[c] \rightarrow P[c]$ and passes turn to Opponent. He will do the same (expand Δ) and the game goes on infinitely.

In the second game $\mathcal{C}_0 = (\varnothing, \{\varphi\}, \{c\})$, where $\varphi = \neg P[c] \rightarrow \neg \exists x P[x]$. φ is an implication, both sending and conclusion of it is not marked, therefore false in the current position. So φ is true, it's Opponent's turn. He expand Δ to $\{c, \alpha\}$ and add to $\mathcal{O}$ formulas $\neg P[c]$, $\exists x P[x]$, $P[\alpha]$. He might not add $\exists x P[x]$ to $\mathcal{O}$ and turn would still be passed to Proponent. But in this case Proponent would have an opportunity to add to $\mathcal{P}$ $\neg \exists x P[x]$ and make this formula true in position (because $\exists x P[x]$ would not be marked), and Opponent still would have needed to add $\exists x P[x]$. After that, Proponent will not be able to pass turn to Opponent, therefore, he will lose.

In the third game let $\mathcal{C}_0 = (\varnothing, \{\varphi\}, \varnothing)$, where $\varphi = \forall x[(P[x] \rightarrow \forall x P[x]) \rightarrow \forall x P[x]] \rightarrow \forall x P[x]$ (*Casari's schema or Casari's formula* [8]). Again φ is an implication, it's Opponent's turn. He needs to make sending false, so he expand Δ and add to $\mathcal{O}$ all formulas $(P[\alpha] \rightarrow \forall x P[x]) \rightarrow \forall x P[x]$ and sending of the φ: $\forall x[(P[x] \rightarrow \forall x P[x]) \rightarrow \forall x P[x]]$. Than Propopent creates mistakes for Opponent by adding to $\mathcal{P}$ all formulas $(P[\alpha] \rightarrow \forall x P[x])$. To get rid of mistakes, Opponent needs to add all $P[\alpha]$, and than Proponent just add to $\mathcal{P}$ $\forall x P[x]$. The only thing Opponent can do now is to expand Δ and repeat everything again. As we can see, this is the winning strategy for Proponent, but φ is not true in all Kripke models. This formula will give us a useful class of Kripke frames (class of all Kripke frames in which Casari's formula is valid; let us call it Casari's class (Kripke frame is from Casari's class iff in every countable sequence of worlds ω_i their individual domains Δ_i remain finite and stabilize; so class of Casari's Kripke frames includes all Noetherian Kripke frames)).

It seems to me that, informally, this game (and Mezhirov's game for propositional intuitionistic logic) could be understood as follows: Opponent is trying to build a theory that belies Proponent's assertion that φ follows from $\mathcal{O}_0$ (or, in the case of $\mathcal{O}_0 = \varnothing$, is trying to build a theory that shows that Proponent's thesis (φ) is not valid in general). And this theory must be coherent (Opponent must have no mistakes), otherwise his approach is considered unsuccessful.

While getting closer to results, I should mention that there was some interest in considering variation of the game with only finite Δ (and Opponent can expand Δ adding only finite number of objects) because of better connection with Kripke models, so there appeared results for two variations of the game (described one (let us call it infinite) and the same but with finite Δ (finite variation)).

Theorem 1. *In the infinite variation, Proponent has a winning strategy in position $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ (with possibly infinite $\mathcal{O}_0$) iff $\mathcal{O}_0 \vDash \varphi$, where $\vDash$ is the entailment in logic of all Noetherian Kripke frames.*

Theorem 2. *In the infinite variation, Proponent has a winning strategy in position $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ (with only finite $\mathcal{O}_0$) iff $\mathcal{O}_0 \vDash \varphi$, where $\vDash$ is the entailment in logic of all Casari's Kripke frames.*

These theorems lead, inter alia, to the fact that logics of Noetherian Kripke frames and of Casari's Kripke frames have the same weak entailment (entailment from finite sets of formulas). Similar results we can see for the finite variation.

Theorem 3. *In the finite variation, Proponent has a winning strategy in position $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ (with possibly infinite $\mathcal{O}_0$, but with only finite Δ_0) iff $\mathcal{O}_0 \vDash \varphi$, where $\vDash$ is the entailment in logic of all Noetherian Kripke frames with only finite individual domains Δ in each world.*

Theorem 4. *In the finite variation, Proponent has a winning strategy in position $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ (with only finite $\mathcal{O}_0$) iff $\mathcal{O}_0 \vDash \varphi$, where $\vDash$ is the entailment in logic of all Casari's Kripke frames with only finite individual domains Δ in each world.*

Theorem 5. *In the finite variation, Proponent has a winning strategy in position $\mathcal{C}_0 = (\mathcal{O}_0, \{\varphi\}, \Delta_0)$ (with only finite $\mathcal{O}_0$) iff $\mathcal{O}_0 \vDash \varphi$, where $\vDash$ is the entailment in logic of all finite Kripke frames with only finite individual domains Δ in each world.*

As we can see, in the case of only finite individual domains in each world, logics of Noetherian Kripke frames, of Casari's Kripke frames and of finite Kripke frames have the same weak entailment.

As I mentioned, the main goal of this study was to find a game with strong connection with Kripke models. Partially, this has been achieved (proofs for all 5 theorems contain building a strategy for Opponent by "walking" from one world of a model to another); in addition, some connections have been established between weak entailment of logics of some classes. But the next step would be to find a triple: a class of Kripke frames, a game semantics and a calculus (probably, an infinitary sequent calculus) with the same strong entailment (entailment from not only finite, but from any sets of formulas). In this case, it is better to take a simpler class of Kripke frames in terms of the possible calculus for this class. So, because of this, Casari's class looks better than the Noetherian class. Therefore, I am trying right now to change rules of the game to get the same strong entailment as in logic of all Kripke frames from Casari's class.

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KRIPKE (IN)COMPLETENESS OF PREDICATE MODAL LOGICS WITH AXIOMS OF BOUNDED ALTERNATIVITY

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1. Introduction and preliminaries

The Kripke frame semantics is a valuable tool for the analysis of propositional modal logics. Despite by now well known phenomenon of propositional Kripke incompleteness, examples of Kripke incomplete propositional modal logics are rather contrived. By contrast, in predicate modal logic, Kripke incompleteness is common, and the precise boundaries of the usefulness of Kripke frame semantics are not as well understood as for propositional logics. At least in part, this situation arises since canonical models for predicate modal logics are not as well-behaved as canonical models for propositional logics. Even in cases when Kripke completeness can be obtained, a suitable Kripke frame is not canonical (in other words, canonicity is not as common in predicate modal logic as it is in propositional modal logic).

Here, we investigate both Kripke completeness and Kripke incompleteness in the context of logics $\mathbf{QAlt}_n$, which are minimal modal predicate logics containing the propositional axiom of bounded alternativity (here, $n \geqslant 1$),

$$\boldsymbol{alt}_n \quad = \quad \neg \bigwedge_{0 \leqslant i \leqslant n} \Diamond (p_i \wedge \bigwedge_{j \neq i} \neg p_j),$$

corresponding to the Kripke frame condition $|R(w)| \leqslant n$ whenever $w \in W$ (‘n-alternativity’), as well as their minimal extensions containing axioms T (‘reflexivity’) and 4 (‘transitivity’). We show, using selective submodels [2], that both $\mathbf{QAlt}_n$ and $\mathbf{QTAlt}_n$ are strongly Kripke complete and, using Kripke bundle semantics [1, Chapter 5], that logics $\mathbf{QK4Alt}_n$ and $\mathbf{QS4Alt}_n$ are Kripke incomplete.[1]

We work with the language containing a countable supply of predicate letters of every arity, Boolean connectives, quantifier symbols, and a unary modal operator $\Box$. The definition of a formula is standard. We also use the abbreviation $\Box^{\leqslant n} A := \bigwedge_{i=0}^{n} \Box^i A$. By a predicate modal logic we mean a set of formulas including the classical predicate logic $\mathbf{QCL}$, the minimal propositional modal logic $\mathbf{K}$, and closed under Substitution, Modus Ponens, Generalization, and Necessitation. If Λ is a propositional modal logic, the minimal predicate modal logic including Λ is denoted by $\mathbf{Q}\Lambda$.

We briefly recall the Kripke frame semantics for predicate logics. A *Kripke frame* is a pair (W, R) where $W \neq \varnothing$ and $R \subseteq W \times W$. A *predicate Kripke frame* is a tuple $\boldsymbol{F} = (F, D)$ where $F = (W, R)$ is a Kripke frame and $D = \{D_u \mid u \in W\}$ is a system of non-empty domains satisfying the condition that $D_u \subseteq D_v$ whenever uRv (‘expanding domains’). A model over a predicate Kripke frame $\boldsymbol{F}$ is a pair $M = (\boldsymbol{F}, \xi)$, where ξ is a family $(\xi_u)_{u \in W}$ of maps such that $\xi_u(P^n) \subseteq D_u^n$, for each n-ary predicate letter P^n. The truth relation between models M, worlds u, and D_u-sentences A (a D_u-sentence is obtained from a formula by substituting elements of D_u for parameters of the formula) is standard; in particular,

- $M, u \models P(a_1, \ldots, a_n)$ if $(a_1, \ldots, a_n) \in \xi_u(P)$;
- $M, u \models \Box A(a_1, \ldots, a_n)$ if $M, v \models A(a_1, \ldots, a_n)$ whenever $v \in R(u)$.

A formula is true in a model if its universal closure is true at every world of the model. A formula is valid on a predicate Kripke frame if it is true in every model over the predicate frame.

If $\mathscr{C}$ is a class of predicate Kripke frames, the set of formulas valid on $\mathscr{C}$ is a modal predicate logic, denoted by $\mathtt{L}(\mathscr{C})$. If there exists a class $\mathscr{C}$ of predicate Kripke frames such that $L = \mathtt{L}(\mathscr{C})$, the logic L is *Kripke complete*; if, in addition, every set of L-consistent formulas is satisfiable in a model over a predicate Kripke frame validating L, then L is *strongly Kripke complete*.

[1] Strong completeness of logics $\mathbf{QAlt}_n$ was claimed, without proof, in [3]; here, we give a detailed proof.

2. Kripke completeness of $\mathbf{QAlt}_n$ and $\mathbf{QTAlt}_n$

Throughout this section, unless stated otherwise, L is a predicate modal logic. For completeness proofs, we use languages extended with a set of constants of arbitrary cardinality. We assume, for now, a fixed universal set $\mathcal{S}$ of constants of infinite cardinality κ. A set $C \subseteq \mathcal{S}$ of constants is *$\mathcal{S}$-small* if $|\mathcal{S} - C| = \kappa$. If C is a set of constants, a C-sentence is a sentence possibly containing constants from C. The set of all C-sentences is denoted by $\mathcal{L}(C)$. A *theory* is a set of C-sentences, for some $C \subseteq \mathcal{S}$. If Γ is a theory, the set of constants occurring in Γ is denoted by C_Γ; the set of all C_Γ-sentences is denoted by $\mathcal{L}(\Gamma)$.

A theory Γ is *Henkin* if, for every sentence $\exists x\, A(x) \in \mathcal{L}(\Gamma)$, there exists $c \in C_\Gamma$ such that $\exists x A(x) \to A(c) \in \Gamma$. A maximal L-consistent theory is called *L-complete*. It can be easily checked that every L-complete Henkin theory Γ has the *existence property*: $\exists x A(x) \in \Gamma \iff (\exists\, c \in C_\Gamma)\, A(c) \in \Gamma$.

Let L be a first-order modal logic. An $(L, \mathcal{S})$-*place* (simply L-*place* if $\mathcal{S}$ is clear from the context or immaterial) is an L-complete Henkin theory with an $\mathcal{S}$-small set of constants.

Lemma 1. *Every L-consistent theory with an $\mathcal{S}$-small set of constants is included into some $(L, \mathcal{S})$-place.*

The *canonical predicate Kripke frame* for L w.r.t. $\mathcal{S}$ is the tuple $F^{\mathcal{S}}_L := (W^{\mathcal{S}}_L, R^{\mathcal{S}}_L, D^{\mathcal{S}}_L)$, where $W^{\mathcal{S}}_L$ is the set of all $(L, \mathcal{S})$-places; $R^{\mathcal{S}}_L$ is the canonical accessibility relation on $W^{\mathcal{S}}_L$ defined as follows: $\Gamma\, R^{\mathcal{S}}_L\, \Delta$ if $\Box^-\Gamma \subseteq \Delta$; and $D^{\mathcal{S}}_L \colon W^{\mathcal{S}}_L \to 2^{\mathcal{S}}$ is the map defined by $D^{\mathcal{S}}_L(\Gamma) = C_\Gamma$. The *canonical Kripke model* for L w.r.t. $\mathcal{S}$ is the tuple $M^{\mathcal{S}}_L := (F^{\mathcal{S}}_L, \xi^{\mathcal{S}}_L)$, where $F^{\mathcal{S}}_L$ is the canonical predicate Kripke frame and $\xi^{\mathcal{S}}_L$ is the canonical valuation defined by$(\xi^{\mathcal{S}}_L)_\Gamma(P^m_k) := \{\mathbf{c} \in C^m_\Gamma \mid P^m_k(\mathbf{c}) \in \Gamma\}$.

Theorem 2. *For every $\Gamma \in W^{\mathcal{S}}_L$ and $A \in \mathcal{L}(C_\Gamma)$,*

$$\mathbf{M}^{\mathcal{S}}_L, \Gamma \models A \iff A \in \Gamma.$$

From now on the universal set of constants $\mathcal{S}$ is no longer fixed; from now on, it is a parameter. A logic L is *canonical* if $F^{\mathcal{S}}_L \models L$, for every universal set $\mathcal{S}$ of constants. As in propositional logic, every canonical logic is strongly Kripke complete, but the examples of predicate canonical logics are scarce (see [1, Section 6.1]). In particular, it can be shown that logics $\mathbf{QAlt}_n$ and $\mathbf{QTAlt}_n$ are not canonical (proof idea: every world Γ containing $\Diamond\top$ in canonical models for these logics sees infinitely many words containing constants outside of C_Γ). Nevertheless, these logics, as we next show, are Kripke complete. To prove this, we use the method of selective submodels [2, Section 6] resembling selective filtration in propositional modal logic and Tarski-Vaught test in classical model theory.

A Kripke model $M' = (W', R', D', \xi')$ is a *weak submodel* of a Kripke model $M = (W, R, D, \xi)$ if $W' \subseteq W$, $R' \subseteq R$, and, for every $w \in W'$, both $D_w = D'_w$ and $\xi'_w = \xi_w$. If, additionally, $M, w \models \Diamond A \implies \exists u \in R'(w)\, M, u \models A$, for every $w \in W'$ and every D_w-sentence A, then M' is a *selective* weak submodel of M.

Lemma 3. *Let $M' = (W', R', D', \xi')$ be a selective weak submodel of $M = (W, R, D, \xi)$. Then $M, w \models A \iff M', w \models A$, for every $w \in W'$ and every D_w-sentence A.*

A *quasi-canonical model* for a logic L is a selective weak submodel of $M^{\mathcal{S}}_L$ (for some $\mathcal{S}$). A logic L is *quasi-canonical* if, for every L-place Γ, there exists a quasi-canonical model over a predicate frame containing Γ and validating L. By Theorem 2 and Lemma 3, if $M' = (W', R', D', \xi')$ a quasi-canonical model for L, then, $M', \Gamma \models A \iff A \in \Gamma$, for every $\Gamma \in W'$. Hence, due to Lemma 1,

Theorem 4. *Every quasi-canonical predicate modal logic is strongly Kripke complete.*

Theorem 5. *Let $L = \mathbf{QAlt}_n$ or $L = \mathbf{QTAlt}_n$, for some $n \geqslant 1$. Then L is quasi-canonical and, hence, strongly Kripke complete.*

Proof. Let $M_L = (W_L, R_L, D_L, \xi_L)$ be a canonical model for L, and let $\Gamma_0 \in W_L$. We obtain a selective submodel M of M_L over a frame validating L and containing Γ_0. First, we prove the following:

Lemma 6. *Let $\Gamma \in W_L$ and $X^\Gamma := \{\Delta \mid \Delta$ is L-complete & $\mathcal{L}(\Delta) = \mathcal{L}(\Gamma)$ & $\Box^-\Gamma \subseteq \Delta\}$. Then $|X^\Gamma| \leqslant n$.*

Proof. Suppose that $\Delta_0, \dots, \Delta_n$ are distinct theories from X^Γ. Since these theories are L-complete and $\mathcal{L}(\Delta_0) = \ldots = \mathcal{L}(\Delta_n) = \mathcal{L}(\Gamma)$, for each $i, j \in \{0, \dots, n\}$ with $i \neq j$, there exists $A_{ij} \in \mathcal{L}(\Gamma)$ such that $A_{ij} \in \Delta_i$, but $A_{ij} \notin \Delta_j$. For every $i \in \{0, \dots, n\}$, let $B_i = \bigwedge\limits_{j \neq i} (A_{ij} \wedge \neg A_{ji})$. Then, $B_i \in \Delta_j$ iff $i = j$. Hence, $\bigwedge\limits_{0 \leqslant i \leqslant n} \Diamond(B_i \wedge \bigwedge\limits_{j \neq i} \neg B_j) \in \Gamma$. But $\vdash_{\mathbf{QAlt}_n} \neg(\bigwedge\limits_{0 \leqslant i \leqslant n} \Diamond(B_i \wedge \bigwedge\limits_{j \neq i} \neg B_j))$. Thus, Γ is L-inconsistent, contrary to the assumption. $\square$

We now proceed with the proof of the theorem, distinguishing two cases.

Case $L = \mathbf{QAlt}_n$: We define the set W of worlds and the accessibility relation R of the model M by recursion. Set $W_0 = \varnothing$, $W_1 = \{\Gamma_0\}$, and $R_0 = R_1 = \varnothing$. Suppose the sets $W_0, \ldots, W_k$ and the relations $R_0, \ldots, R_k$, for some $k < \omega$, have been defined. To define W_{k+1} and R_{k+1}, consider, for each $\Gamma \in W_k - W_{k-1}$, the set X^Γ defined in Lemma 6. By Lemma 6, $|X^\Gamma| \leqslant n$. By Lemma 1, for each $\Delta \in X^\Gamma$, there exists $\Delta' \in W_L$ such that $\Delta \subseteq \Delta'$; let Y^Γ be the set containing exactly one such $\Delta' \in W_L$ for each $\Delta \in X^\Gamma$. Then, $|Y^\Gamma| \leqslant n$. By Existence Lemma and Lindenbaum lemma, for every sentence A, if $\Diamond A \in \Gamma$, then $(\exists \Delta_0 \in X^\Gamma)\, A \in \Delta$. Hence,

$$\Diamond A \in \Gamma \Longrightarrow (\exists \Delta \in Y^\Gamma)\, A \in \Delta. \tag{1}$$

Set $W_{k+1} = W_k \cup \bigcup\limits_{\Gamma \in W_k - W_{k-1}} Y^\Gamma$ and $R_{k+1} = R_k \cup \bigcup\limits_{\Gamma \in W_k - W_{k-1}} \left(\{\Gamma\} \times Y^\Gamma\right)$. As we have seen, if $\Gamma \in W_k - W_{k-1}$, then $|Y^\Gamma| \leqslant n$, and so $|R_{k+1}(\Gamma)| \leqslant n$. Observe that $R_{k+1} \subset R_L$. Lastly, let $W = \bigcup\limits_{k<\omega} W_k$ and $R = \bigcup\limits_{k<\omega} R_k$. Then, by (1),

$$\forall \Gamma \in W\, \forall A \in \mathcal{L}(\Gamma)\, \big(\Diamond A \in \Gamma \Longrightarrow (\exists \Delta \in R(\Gamma))\, A \in \Delta\big). \tag{2}$$

By definition of R and Lemma 6, $|R(\Gamma)| \leqslant n$, for each $\Gamma \in W$. Also, $R \subseteq R_L$. Hence, $(W, R) \models \boldsymbol{alt}_n$. Lastly, let $M := M_L \restriction W$. Then, $(W, R, D) \models L$. Thus, M is a submodel of M_L over an L-frame containing Γ_0. By (2) and Theorem 2, M is a selective submodel of M_L.

Case $L = \mathbf{QTAlt}_n$: The set W and the relation R are again defined by recursion. We set $W_0 = \{\Gamma_0\}$, $R_0 = R_1 = \{(\Gamma_0, \Gamma_0)\}$. We need to make sure that every relation R_k, and hence their union R, is reflexive. Suppose R_k is reflexive, for some $k < \omega$. Since R_L is reflexive, it follows that $\Gamma \in X_\Gamma$. We pick the L-complete set $\Gamma' \in Y^\Gamma$ so that $\Gamma' = \Gamma$. Then, R_{k+1} is reflexive. Hence, R is reflexive, and so and $(W, R, D) \models L$. $\square$

3. Kripke incompleteness of $\mathbf{QK4Alt}_n$ and $\mathbf{QS4Alt}_n$

To prove Kripke incompleteness of logics $\mathbf{QK4Alt}_n$ and $\mathbf{QS4Alt}_n$, we use the semantics of Kripke bundles [1, Chapter 5]. A *Kripke bundle* is a tuple $\mathbb{F} = (F, D, \rho)$, where $F = (W, R)$ is a Kripke frame, $D = \{D_u \mid u \in W\}$ is a family of non-empty disjoint domains, and $\rho = \{\rho_{uv} \mid (u, v) \in R\}$ is a family of inheritance relations $\rho_{uv} \subseteq D_u \times D_v$ satisfying the constraint that $\rho_{uv}(a) \neq \varnothing$ whenever uRv and $a \in D_u$. Models over Kripke bundles are defined analogously to models over Kripke frames. The truth clause for formulas beginning with $\Box$ is as follows: $M, u \models \Box A(a_1, \ldots, a_n)$, with distinct $a_1, \ldots, a_n \in D_u$, if

$$\forall v \in R(u) \forall b_1 \in \rho_{uv}(a_1) \ldots \forall b_n \in \rho_{uv}(a_n)\, M, v \models A(b_1, \ldots, b_n).$$

A formula is *true* in Kripke bundle model if its universal closure is true at every world of the model. A formula A is *strongly valid* in a Kripke bundle $\mathbb{F}$ (notation: $\mathbb{F} \Vdash A$) if every substitution instance of A is true in every model over $\mathbb{F}$. The following is well known [1, Proposition 5.2.12]:

Proposition 7. *Let $\mathbb{F}$ be a Kripke bundle. Then the set $\{A \mid \mathbb{F} \Vdash A\}$ is a modal predicate logic.*

With every Kripke bundle $\mathbb{F} = (W, R, D, \rho)$, we associate a family $\{(W_n, R_n) \mid n < \omega\}$ of Kripke frames: put $D_0 := W$ and $R_0 := R$; put $D_1 := \bigcup\{D_u \mid u \in W\}$ and $R_1 := \bigcup\{\rho_{uv} \mid uRv\}$; for every $n > 1$, put $D_n := \bigcup\{D_u^n \mid u \in W\}$ and

$$R_n \quad := \quad \{(\mathbf{a}, \mathbf{c}) \in D_n \times D_n \mid \forall j\, a_j R_1 b_j \text{ and } \forall j, k\, (a_j = a_k \Rightarrow b_j = b_k)\}.$$

The following is well known [1, Proposition 5.3.7]:

Proposition 8. *Let $\mathbb{F}$ be a Kripke bundle and A a modal propositional formula. Then, $\mathbb{F} \Vdash A$ iff $F_n \models A$, for every $n < \omega$.*

Theorem 9. *Let $L = \mathbf{QK4Alt}_n$ or $L = \mathbf{QS4Alt}_n$, for some $n \geqslant 1$. Then, L is Kripke incomplete.*

To prove incompleteness of $\mathbf{QK4Alt}_n$, we make use of the formula $\forall ref := \forall x\, (\Box P(x) \to P(x))$. We show that every Kripke predicate frame validating $\mathbf{QK4Alt}_n$ validates $A_n := \Diamond^{\leqslant n+1}\top \to \Diamond\forall ref$, but $A_n \notin \mathbf{QK4Alt}_n$.

Suppose that $\boldsymbol{F} = (W, R, D) \models \mathbf{QK4Alt}_n$, and so R is transitive and n-alternative. Let M be a model over $\boldsymbol{F}$ and $u_0 \in W$. Assume that $M, u_0 \models \Diamond^{\leqslant n+1}$. Then, there exist $u_1, \ldots, u_{n+1} \in W$ such that $u_0 R u_1 R \ldots R u_{n+1}$. Since R is n-alternative, there exist $k, j \leqslant n+1$ such that $k \neq j$ and $u_k = u_j$. But then u_k is reflexive, and so $M, u_k \models \forall ref$. Hence, $M, u_0 \models \Diamond\forall ref$ and so $M, u_0 \models A_n$.

To show that $A \notin \mathbf{QK4Alt}_n$, in view of Proposition 7, it suffices to obtain a Kripke bundle strongly validating $\mathbf{QK4Alt}_n$, but refuting A. Define $W = \{u\}$, $R = \{(u,u)\}$, $D_u = \{a,b\}$, and $\rho = \{(a,b),(b,b)\}$. Put $\mathbb{F}_0 = (W,R,D,\rho)$. It should be clear that $\mathbb{F}_0$ is a Kripke bundle. To see that $\mathbb{F}_0 \not\Vdash A$, consider the model $M_0 = (\mathbb{F}_0, \xi)$ with $\xi_u(P) = \{b\}$. Since $M_0, u \models P(b)$, the world u is reflexive, and b is the unique inheritor of a, it follows that $M_0, u \models \Box P(a)$. Since $M_0, u \not\models P(a)$, it follows that $M_0, u \not\models \Box P(a) \to P(a)$ and so $M_0, u \not\models \Diamond\forall \mathit{ref}$. On the other hand, since R is serial, $M_0, u \models \Diamond^{\leqslant n+1}\top$. Hence, $\mathbb{F}_0 \not\models A_n$.

It remains to prove that $\mathbb{F}_0 \Vdash \mathbf{QK4Alt}_n$. We use Proposition 8 to prove that $\mathbb{F}_0 \Vdash \mathbf{QK4Alt}_1$ and hence $\mathbb{F}_0 \Vdash \mathbf{QK4Alt}_n$, for every $n \geqslant 1$. It should be clear that $F_0 = (W,R) \models \mathbf{K4Alt}_1$. Let $n \geqslant 1$ and $\mathbf{d}, \mathbf{e} \in D_n$. Then, $\mathbf{d} R_n \mathbf{e}$ iff $\forall j\, e_j = b$; hence, every $\mathbf{d} \in D_n$ has exactly one R_n-successor, $\mathbf{b}$, and so R_n is transitive and 1-alternative (in fact, functional). Thus, $F_n \models \mathbf{K4Alt}_1$, for every $n < \omega$. Hence, by Proposition 8, $\mathbb{F}_0 \Vdash \mathbf{QK4Alt}_1$ and so $\mathbb{F}_0 \Vdash \mathbf{QK4Alt}_n$, for every $n \geqslant 1$.

The proof for $\mathbf{QS4Alt}_n$ is analogous. Instead of the formula A_n, we use $\Diamond\Box\forall x\,(\Diamond\Box P(x) \to P(x))$, and instead of the Kripke bundle $\mathbb{F}_0$, we use the Kripke bundle $\mathbb{F}_1$ defined as follows: $W = \{u\}$, $R = \{(u,u)\}$, $D_u = \{a,b\}$, $\rho = \{(a,a),(a,b),(b,b)\}$, and $\mathbb{F}_1 = (W,R,D,\rho)$.

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USING THE TEMPORAL MONODIC CLIQUE-GUARDED NEGATION FRAGMENT TO SPECIFY SWARM PROPERTIES

SEN ZHENG, MICHAEL FISHER, AND CLARE DIXON

Abstract. Both the *guarded negation* and the *clique-guarded negation* fragments of first-order logic were shown to be robustly decidable. However, unlike the guarded and the packed fragments, we are not aware of their combination with first-order temporal logics, either considering theoretical properties or their practical application in specifying real-world problems. In this paper, we formally define the monodic clique-guarded negation fragment and explore using this fragment to specify the properties of robot swarms.

1. Introduction

Though First-Order Temporal Logic (FOTL) is very expressive and useful in Computer Science, it is generally highly undecidable and not even recursively enumerable. The seminal work of [6] attempts to identify decidable fragments of FOTL. The idea is to first identify a decidable fragment in the non-temporal part of FOTL, and then extend the fragment with temporal *monodicity*, i.e. each formula in the fragment that is under a temporal operator contains no more than one free variable.

One of the promising decidable fragments in FOTL is the *monodic guarded fragment* (MGF) [6], since the *guarded fragment* [1, 3] is a natural generalisation of modal logics and therefore the fragment inherits desirable computational properties, for example, robust decidability [9], from modal logics. The *packed fragment* [7], the *guarded negation* and the *clique-guarded negation fragments* [2] are decidable fragments that generalise the guarded fragment and inherit its positive properties. However, though the *monodic packed fragment* (MPF) is shown to be decidable [5], yet decidability of the *monodic guarded negation fragment* (MGNF) and the *monodic clique-guarded negation fragment* (MCGNF) is unknown. **Fig. 2** depicts the relationship among these monodic fragments. Due to the decidability result of MGF and MPF, we conjecture that MCGNF is likely to be decidable. As MCGNF subsumes all the aforementioned monodic fragments, this short paper will focus on MCGNF.

We will particularly be concerned with *robot swarms*. A robot swarm is a collection of (often simple) robots designed to work together to carry out tasks. Such swarms rely on the simplicity of the individual robots, the fault tolerance inherent in having a large population of often identical robots and the self-organised behaviour of the swarm as a whole. An overview of swarm robotics algorithms can be found in [8]. With such multiple-entity systems, verification that desired properties do hold is challenging as the state space becomes large once the number of entities grows. Also with approaches such as model checking or propositional temporal logic, we usually have to fix the number of entities we consider. First-order temporal logic, using a suitable decidable fragment, avoids this need by allowing quantification over the robots. Verification of protocols for multiple entities using monodic first-order temporal logic is described for example in [4]. This short paper explores how MCGNF can specify the properties of swarms.

2. Temporal monodic clique-guarded negation fragment

We now formally define MCGNF. Formulas in MCGNF are interpreted in the standard first-order temporal structure, where a strict linear order represents the flow of time.

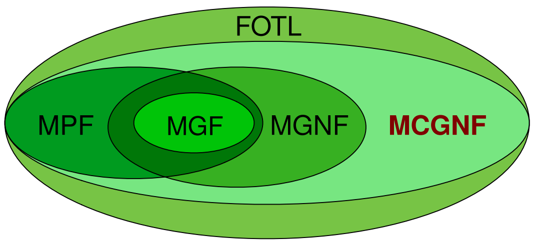

Figure 2. The relationship of the monodic guarded fragments and FOTL

Definition 1. *The* temporal monodic clique-guarded negation fragment (MCGNF) *is a fragment of temporal first-order logic without (non-constant) function symbols but with equality, inductively defined as follows:*

(1) $\top$ *and* $\bot$ *belong to MCGNF.*
(2) *If A is an atom, then A belongs to MCGNF.*
(3) *If A and B are atoms, $A \vee B$ and $A \wedge B$ belong to MCGNF.*
(4) *If F is a non-temporal formula in MCGNF, then $\exists \overline{x} F$ belongs to MCGNF.*
(5) *Let F be a formula in MCGNF and $\mathbb{G}(\overline{x}, \overline{y})$ a conjunction of atoms. Then, $\exists \overline{x} \mathbb{G}(\overline{x}, \overline{y}) \wedge \neg F$ belongs to MCGNF if*
   (a) *all free variables of F are in $\overline{y}$,*
   (b) *each variable in $\overline{x}$ occurs in at most one atom of $\mathbb{G}(\overline{x}, \overline{y})$ if $\overline{x}$ exist,*
   (c) *each pair of distinct variables in $\overline{y}$ co-occurs in at least one atom of $\mathbb{G}(\overline{x}, \overline{y})$.*
(6) *If F belongs to MCGNF and F contains at most one free variable, then $\bigcirc F$, $\Box F$ and $\Diamond F$ belong to MCGNF.*

We will focus on closed formulas in MCGNF, and we use notation a, b and c to denote constants.

3. Specifying robot swarm properties

We use MCGNF to specify three swarm properties. The first two relate to the "coherence" property, namely robots maintaining a connected group, described for example in [10]. The third example relates to the shape of the robot swarm and is inspired by robots having to form particular shapes such as lines or squares (see for example [8]) where a line of robots might be needed to enter a pipe for inspection or to form a communication network while a square might be needed for object transportation.

Specifying a clique of robots. MCGNF can describe robots that form a clique. For example, a clique of four distinctive robots can be specified using the monodic clique-guarded negation formula

$$\exists x_{1\ldots4}(R(x_1, x_2) \wedge R(x_1, x_3) \wedge R(x_1, x_4) \wedge R(x_2, x_3) \wedge R(x_2, x_4) \wedge R(x_3, x_4)\wedge \\ x_1 \not\approx x_2 \wedge x_1 \not\approx x_3 \wedge x_1 \not\approx x_4 \wedge x_2 \not\approx x_3 \wedge x_2 \not\approx x_4 \wedge x_3 \not\approx x_4).$$

Specifying a robot leaving a robot clique. The following formula in MCGNF describes that a robot a leaves an a-containing three-node clique in the next temporal step:

$$\exists x_{1\ldots2}(\mathit{adjacent}(x_1, x_2) \wedge \mathit{adjacent}(x_2, a) \wedge \mathit{adjacent}(x_1, a)) \rightarrow \\ \bigcirc \exists y_{1\ldots2}(\mathit{adjacent}(y_1, y_2) \wedge \neg \mathit{adjacent}(y_1, a) \wedge \neg \mathit{adjacent}(y_2, a)).$$

MCGNF can also describe that in the next step, a robot in a clique connects to only one robot in that clique. This is specified as follows:

$$\exists x_{1\ldots3}(\mathit{connect}(x_1, x_2) \wedge \mathit{connect}(x_1, x_3) \wedge \mathit{connect}(x_1, a) \wedge \mathit{connect}(x_2, x_3) \\ \wedge \mathit{connect}(x_2, a) \wedge \mathit{connect}(x_3, a)) \rightarrow \bigcirc \exists y_{1\ldots3}(\mathit{connect}(y_1, y_2) \wedge \mathit{connect}(y_1, y_3) \\ \wedge \mathit{connect}(y_2, y_3) \wedge \mathit{connect}(a, y_2) \wedge \neg \mathit{connect}(a, y_1) \wedge \neg \mathit{connect}(a, y_3)).$$

Fig. 3 depicts the processes relating to the above monodic clique-guarded negation formula.

Specifying shapes of robots. A line of three robots can be specified using the following monodic clique-guarded formula:

$$\mathit{adjacent}(a, b) \wedge \mathit{adjacent}(b, c) \wedge \neg \exists x(\mathit{adjacent}(x, b) \wedge x \not\approx a \wedge x \not\approx c)\wedge \\ \neg \exists x(\mathit{adjacent}(x, a) \wedge x \not\approx b) \wedge \neg \exists x(\mathit{adjacent}(x, c) \wedge x \not\approx b).$$

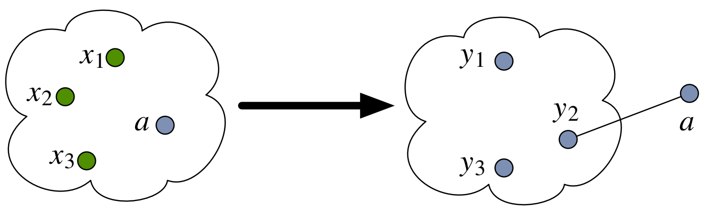

Figure 3. A robot leaves a clique and connects to only one node in the clique

FIGURE 4. Line-shaped (left) and quadrilateral-shaped (right) swarms

Following a similar construction, we can use the monodic clique-guarded formula

$$\begin{aligned}&adjacent(a,b) \wedge adjacent(b,c) \wedge adjacent(c,d) \wedge adjacent(d,a) \wedge \\ &\neg\exists x(adjacent(x,a) \wedge x \not\approx b \wedge x \not\approx d) \wedge \neg\exists x(adjacent(x,b) \wedge x \not\approx a \wedge x \not\approx c) \wedge \\ &\neg\exists x(adjacent(x,c) \wedge x \not\approx b \wedge x \not\approx d) \wedge \neg\exists x(adjacent(x,d) \wedge x \not\approx a \wedge x \not\approx c)\end{aligned}$$

to describe four robots forming a quadrilateral. **Fig. 4**, from left to right, depicts robots forming a line and a quadrilateral, respectively.

4. Conclusion

We have applied MCGNF to formalised three use-cases of swarm robots. It gives us confidence that MCGNF can be useful in specifying more complex properties that previously cannot be specified solely using propositional temporal logic or MPF. One of our future challenges is to handle negated formulas in MCGNF, since the free variables of these formulas need to occur in an atom or a clique of atoms, which is sometimes not guaranteed when specifying swarm properties.

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