Using the temporal monodic clique-guarded negation fragment to specify swarm properties

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Abstract. Both the *guarded negation* and the *clique-guarded negation* fragments of first-order logic were shown to be robustly decidable. However, unlike the guarded and the packed fragments, we are not aware of their combination with first-order temporal logics, either considering theoretical properties or their practical application in specifying real-world problems. In this paper, we formally define the monodic clique-guarded negation fragment and explore using this fragment to specify the properties of robot swarms.

Keywords: temporal logic · first-order fragments · formal verification

1 Introduction

Though First-Order Temporal Logic (FOTL) is very expressive and useful in Computer Science, it is undecidable and not even recursively enumerable. The seminal work of [6] attempts to identify decidable fragments of FOTL. The idea is to first identify a decidable fragment in the non-temporal part of FOTL, and then extend the fragment with temporal *monodicity*, i.e. each formula in the fragment that is under a temporal operator contains no more than one free variable.

One of the promising decidable fragments in FOTL is the *monodic guarded fragment* (MGF) [6], since the *guarded fragment* [1, 3] is a natural generalisation of modal logics and therefore the fragment inherits desirable computational properties, for example, robust decidability [9], from modal logics. The *packed fragment* [7], the *guarded negation* and the *clique-guarded negation fragments* [2] are decidable fragments that generalise the guarded fragment and inherit its

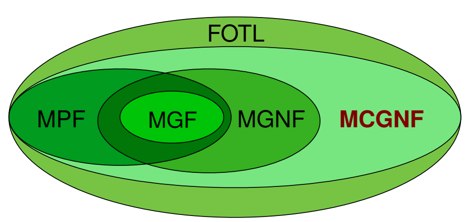

Fig. 1. The relationship of the monodic guarded fragments and FOTL

positive properties. However, though the *monodic packed fragment* (MPF) is shown to be decidable [5], the decidability of the *monodic guarded negation fragment* (MGNF) and the *monodic clique-guarded negation fragment* (MCGNF) is unknown. **Fig. 1** depicts the relationship among these monodic fragments. Due to the decidability result of MGF and MPF, we conjecture that MGNF and MCGNF are likely to be decidable. This makes applying deductive methods to solve MCGNF (and its subfragments) satisfiablity checking problems possible. As MCGNF subsumes all the aforementioned monodic fragments, this short paper will focus on MCGNF and its subfragments.

We particularly concern with *robot swarms*. A robot swarm is a collection of (often simple) robots designed to work together to carry out tasks. Such swarms rely on the simplicity of the individual robots, the fault tolerance inherent in having a large population of often identical robots and the self-organised behaviour of the swarm as a whole. An overview of swarm robotics algorithms can be found in [8]. With such multiple-entity systems, verification that desired properties do hold is challenging as the state space becomes large once the number of entities grows. Also like approaches such as model checking or propositional temporal logic, we usually have to fix the number of entities we consider. First-order temporal logic, using a suitable decidable fragment, avoids this need by allowing quantification over the robots. Verification of protocols for multiple entities using monodic first-order temporal logic is described for example in [4]. This short paper explores how MCGNF and its subfragments can specify the properties of swarms.

2 Temporal monodic clique-guarded negation fragment

We now formally define MCGNF. Formulas in MCGNF are interpreted in the standard first-order temporal structure, where a strict linear order represents the flow of time.

Definition 1. *The* temporal monodic clique-guarded negation fragment (MCGNF) *is a fragment of temporal first-order logic without (non-constant) function symbols but with equality, inductively defined as follows:*

1. *$\top$ and $\bot$ belong to MCGNF.*
2. *If A is an atom, then A belongs to MCGNF.*
3. *If A and B are atoms, $A \vee B$ and $A \wedge B$ belong to MCGNF.*
4. *If F is a non-temporal formula in MCGNF, then $\exists \overline{x} F$ belongs to MCGNF.*
5. *Let F be a formula in MCGNF and $\mathbb{G}(\overline{x}, \overline{y})$ a conjunction of atoms. Then, $\exists \overline{x} \mathbb{G}(\overline{x}, \overline{y}) \wedge \neg F$ belongs to MCGNF if*
   (a) *all free variables of F are in $\overline{y}$,*
   (b) *$\overline{x}$ does not occur in F and each variable in $\overline{x}$ occurs in at most one atom of $\mathbb{G}(\overline{x}, \overline{y})$, if $\overline{x}$ exist,*
   (c) *each pair of distinct variables in $\overline{y}$ co-occurs in at least one atom of $\mathbb{G}(\overline{x}, \overline{y})$.*
6. *If F belongs to MCGNF and F contains at most one free variable, then $\bigcirc F$, $\Box F$ and $\Diamond F$ belong to MCGNF.*

We will focus on closed formulas in MCGNF, and we use the notation a, b and c to denote constants.

3 Specifying robot swarm properties

We use MCGNF to specify three swarm properties. The first two relate to the "coherence" property, namely robots maintaining a connected group, described for example in [10]. The third example relates to the shape of the robot swarm and is inspired by robots having to form particular shapes such as lines or squares (see for example [8]) where a line of robots might be needed to enter a pipe for inspection or to form a communication network while a square might be needed for object transportation. We use $adj(x,y)$ to denote that x and y are in a certain range and use $con(x,y)$ to denote that x and y can detect each other.

Specifying a clique of robots. MCGNF can describe robots that form a clique. For example, a clique of four distinctive robots can be specified using the monodic clique-guarded negation formula

$$\exists x_{1\dots4}(adj(x_1,x_2) \land adj(x_1,x_3) \land adj(x_1,x_4) \land adj(x_2,x_3) \land adj(x_2,x_4) \land \\ adj(x_3,x_4) \land x_1 \not\approx x_2 \land x_1 \not\approx x_3 \land x_1 \not\approx x_4 \land x_2 \not\approx x_3 \land x_2 \not\approx x_4 \land x_3 \not\approx x_4).$$

Specifying a robot leaving a robot clique. The following formula in MCGNF describes that a robot a leaves an a-containing three-node clique in the next temporal step:

$$\exists x_{1\dots2}(adj(x_1,x_2) \land adj(x_2,a) \land adj(x_1,a)) \rightarrow \\ \bigcirc\, \exists y_{1\dots2}(adj(y_1,y_2) \land \neg adj(y_1,a) \land \neg adj(y_2,a)).$$

MCGNF can also describe that in the next step, a robot in a clique connects to only one robot in that clique. This is specified as follows:

$$\exists x_{1\dots3}(con(x_1,x_2) \land con(x_1,x_3) \land con(x_1,a) \land con(x_2,x_3) \\ \land con(x_2,a) \land con(x_3,a)) \rightarrow \bigcirc \exists y_{1\dots3}(con(y_1,y_2) \land con(y_1,y_3) \\ \land con(y_2,y_3) \land con(a,y_2) \land \neg con(a,y_1) \land \neg con(a,y_3)).$$

Fig. 2 depicts the processes relating to the above monodic clique-guarded negation formula.

Specifying shapes of robots. A line of three robots can be specified using the following monodic clique-guarded formula:

$$adj(a,b) \land adj(b,c) \land \neg\exists x(adj(x,b) \land x \not\approx a \land x \not\approx c) \land \\ \neg\exists x(adj(x,a) \land x \not\approx b) \land \neg\exists x(adj(x,c) \land x \not\approx b).$$

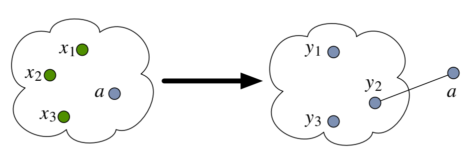

Fig. 2. A robot leaves a clique and connects to only one node in the clique

Fig. 3. Line-shaped (left) and quadrilateral-shaped (right) swarms

Following a similar construction, we can use the monodic clique-guarded formula

$$\begin{aligned}&adj(a,b) \wedge adj(b,c) \wedge adj(c,d) \wedge adj(d,a) \wedge \\ &\neg\exists x(adj(x,a) \wedge x \not\approx b \wedge x \not\approx d) \wedge \neg\exists x(adj(x,b) \wedge x \not\approx a \wedge x \not\approx c) \wedge \\ &\neg\exists x(adj(x,c) \wedge x \not\approx b \wedge x \not\approx d) \wedge \neg\exists x(adj(x,d) \wedge x \not\approx a \wedge x \not\approx c)\end{aligned}$$

to describe four robots forming a quadrilateral. **Fig. 3**, from left to right, depicts robots forming a line and a quadrilateral, respectively.

4 Conclusion

We have applied MCGNF to formalised three use cases of swarm robots. It gives us confidence that MCGNF can be useful in specifying more complex properties that previously cannot be specified using propositional temporal logic or MPF. For example, unlike MPF, MCGNF can express $\neg\exists\overline{x}\phi(\overline{x})$ where $\phi(\overline{x})$ contains only atoms. One of our future challenges is to handle negated formulas in MCGNF, since the free variables of these formulas need to occur in an atom or a clique of atoms, which is sometimes not guaranteed when specifying swarm properties.

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